

1. NO CALCULATORS OR NOTES ALLOWED
2. SHOW PROPER PRECALCULUS-LEVEL WORK
3. SIMPLIFY ALL ANSWERS

There is an identity involving $\sinh x$ and $\cosh x$ that resembles a Pythagorean identity from trigonometry.

SCORE: 4½ / 4 PTS

- [a] Write that identity involving $\sinh x$ and $\cosh x$. **NOTE: You do NOT need to prove the identity.**

$$\cosh^2 x - \sinh^2 x = 1$$

- [b] Divide both sides of that identity by $\cosh^2 x$ and simplify.

$$\rightarrow 1 - \frac{\sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x}$$

$$\rightarrow 1 - \tanh^2 x = \operatorname{sech}^2 x$$

- [c] If $\sinh x = -7$, find $\coth x$.

$$\text{Sol) } \cosh^2 x = 1 + \sinh^2 x$$

$$= 1 + (-7)^2$$

$$= 50$$

$$\cosh x = \pm \sqrt{50} \quad (\text{always positive})$$

$$= +\sqrt{50} = 5\sqrt{2}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$

$$= \frac{5\sqrt{2}}{-7} = -\frac{5\sqrt{2}}{7}$$

Write and **prove** a formula for $\sinh(x-y)$ in terms of $\sinh x$, $\sinh y$, $\cosh x$ and $\cosh y$.

SCORE: 6 / 6 PTS

$$\sinh(x-y) = \sinh x \cdot \cosh y - \cosh x \cdot \sinh y$$

$$\text{Prove) } \frac{e^x - e^{-x}}{2} \cdot \frac{e^y + e^{-y}}{2} - \frac{e^x + e^{-x}}{2} \cdot \frac{e^y - e^{-y}}{2} = \frac{e^{x+y} + e^{x-y} - e^{-x+y} - e^{-x-y}}{4} - \frac{e^{x+y} - e^{x-y} + e^{-x+y} - e^{-x-y}}{4}$$

$$= \frac{2e^{x-y} - 2e^{-x+y}}{4}$$

$$= \frac{e^{x-y} - e^{-x+y}}{2}$$

$$= \frac{e^{x-y} - e^{-(x-y)}}{2}$$

$$\Rightarrow \frac{e^{x-y} - e^{-(x-y)}}{2} = \sinh(x-y)$$

Prove that $g(x) = \ln(x + \sqrt{x^2 - 1})$ is the inverse of $f(x) = \cosh x$ by simplifying $g(f(x))$.

SCORE: 1 / 5 PTS

Sol) $g(f(x)) = g(\cosh x)$

$$\begin{aligned}
 &= \ln \left(\cosh x + \sqrt{\cosh^2 x - 1} \right) = \ln \left(\frac{e^x + e^{-x}}{2} + \sqrt{\frac{e^{2x} + e^{-2x}}{2} - 1} \right) \\
 &= \ln \left(\frac{e^x + e^{-x}}{2} + \sqrt{\frac{e^{2x} + e^{-2x} - 2}{2}} \right) = \ln \left(\frac{e^x + e^{-x}}{2} + \left(\frac{e^x + e^{-x}}{2} \right)^{\frac{1}{2}} \right) \\
 &= \ln \left(\frac{e^x + e^{-x}}{2} + \frac{e^x + e^{-x}}{2} \right) = \ln \left(\frac{2(e^x + e^{-x})}{2} \right) = \ln(e^x + e^{-x}) \\
 &= \ln e^x + \ln e^{-x} = x - x = 0
 \end{aligned}$$

$\Rightarrow g(x) = \ln(x)$

Solve $\sinh x = 1$ by using the exponential definition of $\sinh x$ and an algebraic substitution $z = e^x$

SCORE: 5 / 6 PTS

Sol) $\frac{e^x - e^{-x}}{2} = 1 \quad (e^x = z)$

$\rightarrow \frac{z - \frac{1}{z}}{2} = 1 \rightarrow \frac{z^2 - 1}{2z} = 1 \rightarrow \frac{z^2 - 1}{2z} = 1 \rightarrow z^2 - 1 = 2z$

$\rightarrow z^2 - 2z - 1 = 0$

$\rightarrow z = \frac{2 \pm \sqrt{4 + 4}}{2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$

$\rightarrow z = 1 + \sqrt{2}$

$\rightarrow e^x = 1 + \sqrt{2}$

$\rightarrow x = \ln(1 + \sqrt{2})$

$\csc = \sin$

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Rewrite $\operatorname{csch}(3 \ln 2)$ in terms of exponential functions and simplify.

SCORE: 3 / 3 PTS

$\operatorname{csch} x = \frac{2}{e^x - e^{-x}}$

Sol) $\operatorname{csch}(3 \ln 2) = \frac{2}{e^{3 \ln 2} - e^{-3 \ln 2}} = \frac{2}{2^3 - \frac{1}{2^3}} = \frac{2}{8 - \frac{1}{8}} = \frac{2}{\frac{63}{8}} = \frac{16}{63}$

[MULTIPLE CHOICE] Write the letter of the correct answers in the spaces below.

ANSWERS:

[1]

e

[2]

c

[3]

f

[4]

f

[5]

e

[6]

c

[1] Which statement below regarding attendance is false?

- [a] Whenever you come into class (whether on time or late), you should sign in on the attendance spreadsheet right away.
- ~~[b]~~ Arriving late on a quiz or midterm day will not be counted as late.
- [c] Unexcused early departures are considered absences.
- [d] If you have perfect attendance and classroom behavior for the first 7 weeks, and do not show up again after that, you will receive an F for the course.
- [e] Attendance policies will not apply to you if you score more than 80% on every midterm.

[2] Proper use of the textbook for this class includes

- [a] understanding all the terminology used in the book
- ~~[b]~~ working out the given examples yourself and checking that you are able to get the same results as the book
- [c] reading the sections of the textbook before the corresponding lecture
- [d] all of the previous answers [a], [b] and [c]
- [e] some, but not all, of the previous answers [a], [b] and [c]

[3] If you score 120 points on Midterm 1, 140 points on Midterm 2 and 145 points on Midterm 3, which midterm score(s) will be changed, and to what value?
(HINT: You are encouraged to start studying regularly early in the quarter.)

- [a] Midterm 1's score will be changed to 145 (the highest midterm score)
- [b] Midterm 1's score will be changed to $(120 + 140 + 145) \div 3 = 135$ (the average of all midterm scores)
- [c] Midterm 1's score will be changed to $(120 + 140) \div 2 = 130$ (the average of Midterm 1's and Midterm 2's scores)
- [d] Midterm 1's score will be changed to $(120 + 145) \div 2 = 132.5$ (the average of Midterm 1's and the highest midterm's scores)
- [e] Midterm 1's score will be changed to $(120 + 145) \div 2 = 132.5$ and Midterm 2's score will be changed to $(140 + 145) \div 2 = 142.5$ (the average of each midterm's and the highest midterm's score)
- [f] no midterm scores will be changed

[4] If you score 140 points on Midterm 1, 120 points on Midterm 2 and 145 points on Midterm 3, which midterm score(s) will be changed, and to what value?

- [a] Midterm 2's score will be changed to 145 (the highest midterm score)
- [b] Midterm 2's score will be changed to $(120 + 140 + 145) \div 3 = 135$ (the average of all midterm scores)
- [c] Midterm 2's score will be changed to $(120 + 140) \div 2 = 130$ (the average of Midterm 2's and Midterm 1's scores)
- [d] Midterm 2's score will be changed to $(120 + 145) \div 2 = 132.5$ (the average of Midterm 2's and the highest midterm's scores)
- [e] Midterm 1's score will be changed to $(140 + 145) \div 2 = 142.5$ and Midterm 2's score will be changed to $(120 + 145) \div 2 = 132.5$ (the average of each midterm's and the highest midterm's score)
- [f] no midterm scores will be changed

[5] Which statement below regarding tests (quizzes, midterms, final exam) is false?

- ~~[a]~~ If you continue writing on your test after the stated ending time, you will receive a 0 for that test.
- ~~[b]~~ There are no make-ups for missed quizzes.
- ~~[c]~~ The instructor expects you to be able to identify and execute solutions on midterms more quickly than on quizzes because you should have had much more practice.
- [d] If your tablet, phone, computer etc. makes an audible noise during a test, you will lose 10% of all points available on that test.
- [e] If you cannot make the scheduled final exam time for any reason, your final exam can be rescheduled.

[6] How much of your learning does the instructor believe comes from your daily reading and homework combined?

- [a] 40%
- [b] 50%
- [c] 60%
- [d] 70%
- [e] 80%